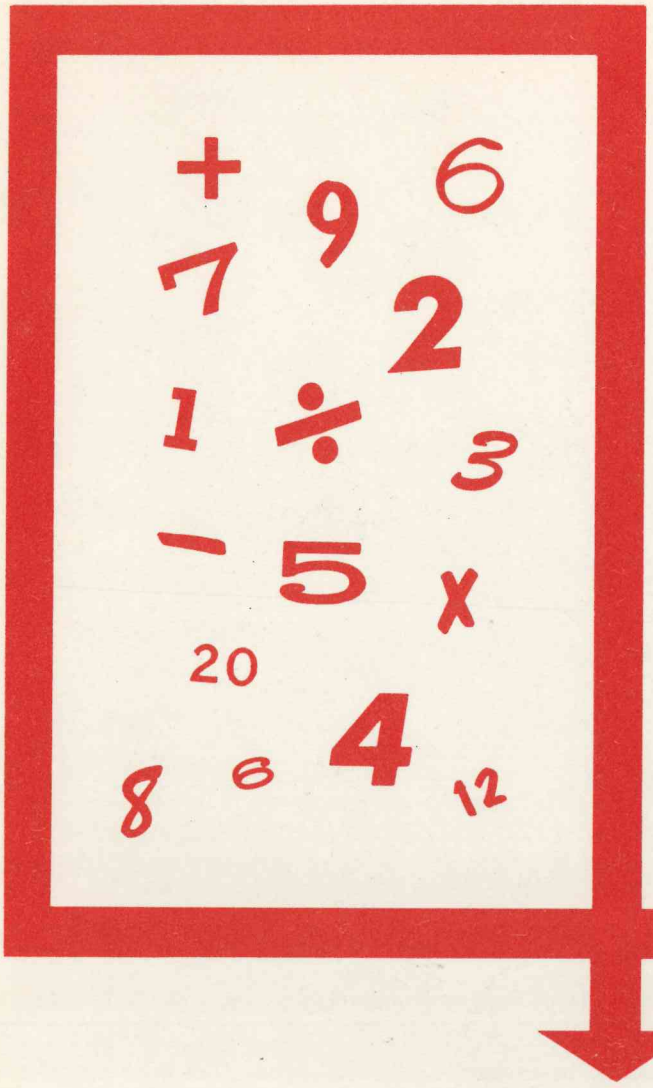

ICS

Practical Arithmetic

Decimals

By

W. M. SPANGLER, M. A.

6750C

Part 3

Edition 1

Practical Arithmetic

PART 3

Decimals

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What This Text Covers . . .

1. Decimals and Their Uses ..... Pages 1 to 11

The importance of decimals in mathematical calculations is discussed in this section. Here you'll learn how to read and write decimals, how to change a decimal to a common fraction, and how to point off a decimal. Mixed decimals are also considered.

2. Addition and Subtraction of Decimals ..... Pages 12 to 18

In this section you'll learn how to add and subtract combinations of decimals, mixed decimals, and whole numbers. Remember that for both addition and subtraction the decimals must be arranged so that the decimal points are in a vertical line.

3. Multiplication of Decimals ..... Pages 19 to 22

The procedure for multiplying decimals is the same as for multiplying whole numbers. In multiplication, the decimal point is not considered until the product of the factors has been found.

4. Division of Decimals ..... Pages 23 to 28

In the division of decimals, as in the other basic operations involving decimals, the only new idea is the position of the decimal point. Remember that the divisor must always be changed to an integer before dividing. You'll learn what to do when there is a remainder and when division does not end.

5. More Facts About Decimals ..... Pages 29 to 36

Here repeating decimals and circulating decimals are discussed. And here you'll learn how to round off a decimal to a desired number of decimal places. The rounding off of whole numbers also enters into the discussion. The method of changing a decimal to a common fraction is covered here.

Practical Arithmetic

PART 3

Decimals

What You Should Know About Decimals

Decimal Fractions

1. In Part 2 of this text series you learned about fractions. We pointed out then that fractions came into use when two numbers were divided and the division didn't work out evenly. We also pointed out that there are two kinds of fractions, common fractions and decimal fractions. You've already learned about common fractions; now you're going to study decimal fractions, or decimals as they're usually called.

In almost all mathematical calculations it's more convenient to use decimals than fractions. In general, the use of decimals greatly simplifies our work in addition, subtraction, multiplication, and division. But what exactly is a decimal, or decimal fraction? Well, first of all it *is* a fraction, and it's less than one. Yet unlike a common fraction, a decimal fraction has no *written* denominator. And it can be identified by a period placed in front of the number. Thus .36, .0215, and .374 are decimals.

The Decimal Point and Decimal Places

2. The period (.), or point, that identifies a decimal is called a *decimal point*. It is of the utmost importance because the value of the decimal depends entirely on the position of the decimal point. Each digit to the *right* of the decimal point occupies a *decimal place*. The number of decimal places in a decimal fraction tells us the value of the unwritten denominator. We count these decimal places and say that .35, for instance, has two places and .096 has three places.

Check Your Learning

You became acquainted with Check Your Learning in Parts 1 and 2 of the *Practical Arithmetic* series. In this text you'll find more programmed statements to help you master decimals.

As before, the statements are numbered consecutively throughout the text. You'll again find the correct answer slightly below and to the right of each statement. Cover our answer with a piece of paper, and complete the statement by writing the best answer you can in the space provided. Then compare your answer with ours. If your answer is correct, go on to the next statement; if your answer is not correct, review that section of the text on which the statement is based.

1. There are two kinds of fractions, _____ fractions and _____ fractions.

(common) (decimal)

2. Decimal fractions are commonly referred to as _____.

(decimals)

3. The value of a decimal is always (more, less) than 1.

(less)

4. Unlike a common fraction, a decimal does not have a written _____.

(denominator)

5. A decimal always has a _____ before it.

(period)

6. The period which is placed before a decimal is called a _____ _____.

(decimal point)

7. Every digit to the (left, right) of the decimal point occupies a decimal place.

(right)

8. In the decimal .382 there are _____ decimal places.

(three)

Writing a Decimal as a Common Fraction

3. Now we're going to write a decimal as a common fraction with a numerator

on the top and a denominator on the bottom, and, of course, a line between. The numerator of our common fraction is the number that follows the decimal point. The denominator is the figure 1 followed by as many zeros as there are places in the decimal. Let's take the decimal .36 as an example. Since 36 follows the decimal point, 36 is the numerator. The decimal has two places, and so the denominator is 1 followed by two zeros, or 100. Now we can write the decimal .36 as the common fraction $\frac{36}{100}$, and we read it as thirty-six hundredths. Let's try a few more: the decimal .5 may be written as $\frac{5}{10}$; $.456 = \frac{456}{1000}$; $.1291 = \frac{1291}{10000}$. All these examples show you that the numerator is the number following the decimal point, and that the denominator is the figure 1 followed by as many zeros as there are places in the decimal.

Check Your Learning

9. A decimal (can, cannot) be written as a common fraction.

(can)

10. When a decimal is written as a common fraction, the number of places in the decimal indicates the value of the _____ of the fraction.

(denominator)

11. To change a decimal to a common fraction, first write the digits following the decimal point as the _____ of the fraction.

(numerator)

12. When a decimal is written as a common fraction, the denominator of the fraction is equal to 1 followed by as many _____ as there are places in the decimal.

(zeros)

13. If the decimal .27 is written as a common fraction, the numerator of the fraction will be _____.

(27)

14. If the decimal .27 is written as a common fraction, the denominator of the fraction will be _____.

(100)

15. Changing the decimal .8 to a common fraction, we have _____.

($\frac{8}{10}$)

Practice Problems

Here are some problems for you to try yourself. Work them out carefully. Then check your results against the answers given at the back of the text.

Write the following decimals as common fractions.

1. .3	5. .304	9. .2843
2. .8	6. .225	10. .5698
3. .14	7. .349	11. .25946
4. .816	8. .4125	12. .309468

Value of the Denominator

4. Now you have seen for yourself that the value of the denominator of a common fraction depends upon the *number of decimal* places in the corresponding decimal. Let's take a closer look at a series of decimals, all having the same numerator, 7.

Decimals	Number of Places	Common Fractions	How to Read
.7	one	$\frac{7}{10}$	7 tenths
.07	two	$\frac{7}{100}$	7 hundredths
.007	three	$\frac{7}{1000}$	7 thousandths
.0007	four	$\frac{7}{10000}$	7 ten-thousandths
.00007	five	$\frac{7}{100000}$	7 hundred-thousandths
.000007	six	$\frac{7}{1000000}$	7 millionths

Notice especially that each of the denominators ends in *ths*; thus you have tenths, hundredths, thousandths, and so on. Actually, it's easy to read decimals. There are only two things to remember:

1) Read the decimal as if it were a whole number.

2) Name the place of the last digit, being sure to end it with *ths*, so that it is clear that the number is a decimal.

Now we'll take a few more decimals, write them as common fractions, and then read them.

$.659 = \frac{659}{1000}$ = six hundred fifty-nine thousandths
$.8056 = \frac{8056}{10000}$ = eight thousand fifty-six ten-thousandths
$.05643 = \frac{5643}{100000}$ = five thousand six hundred forty-three hundred-thousandths
$.000456 = \frac{456}{1000000}$ = four hundred fifty-six millionths

This might be a good time to summarize everything you've learned about decimals into a definition. Let's say that *a decimal is a fraction with an unwritten denominator, preceded by a decimal point whose position indicates the value of the denominator.*

Practice Problems

Write the following decimals, first as common fractions and then in words.

1. .8
2. .25
3. .625
4. .4876
5. .25843
6. .873245

Mixed Decimals

5. Very often we have digits on both sides of the decimal point, as in 19.25, 6.305, and 4658.2. Such numbers are mixed numbers because each one is made up of a whole number (the part before the decimal point) and a fraction (the part after the decimal point). However, since we've already designated numbers consisting of a whole number and a common fraction as mixed numbers, we'll refer to this particular kind of mixed number as a *mixed decimal.* Thus the numbers $1\frac{3}{4}$, $21\frac{1}{2}$, and $32\frac{3}{7}$ will be called mixed numbers, while the numbers 1.75, 10.38, and 32.0 will be called mixed decimals.

Now let's go back to the numbers 19.25, 6.305, and 4658.2. These numbers are read as follows: 19.25 = nineteen *and* twenty-five hundredths; 6.305 = six *and*

three hundred five thousandths; 4658.2 = four thousand six hundred fifty-eight *and* two tenths. Notice that the decimal point is read as *and.* (Note also that since you're now well used to reading numbers, we're going to follow the general practice of omitting the comma in four-digit numbers. Thus instead of 4,658.2, we use 4658.2.)

These numbers may be read another way: 19.25 may be read nineteen *point* twenty-five; 6.305 may be read six *point* three hundred five; 4658.2 may be read four thousand six hundred fifty-eight *point* two.

Sometimes you'll see decimals written with a zero to the left of the decimal point, as 0.5. This zero is used to show that there is *no* whole number. This decimal is simply read five tenths.

Check Your Learning

16. The decimal .009 is read as nine _____ .
(thousandths)

17. When there are digits on both sides of the decimal point, the number is called a _____ decimal.
(mixed)

18. A mixed decimal is made up of a _____ _____ and a decimal.
(whole number)

19. In a mixed decimal, the whole number is the part (before, after) the decimal point.
(before)

20. In the mixed decimal 368.25, the whole number is _____ .
(368)

21. In a mixed decimal, the decimal point is read as _____ or _____ .
(and) (point)

22. The mixed decimal 2.16 is read as two and sixteen _____ .
(hundredths)

23. The number 3.5 is read as three and five tenths, or three _____ five.
(point)

Names of Decimal Places

6. Here is a diagram which will help you understand the relation of decimals and whole numbers in case it isn't yet completely clear to you.

Using the *units place* as the starting point, you can see that whole numbers *increase* to the left, while decimals *decrease* to the right. The digits on both sides

Hundreds of millions	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths	Hundred-millionths
4	7	6,	5	4	9,	2	1	8	.	7	3	3	4	9	8	7	6

of the decimal point correspond in name, except that the decimals end in *ths* to distinguish them from the whole numbers.

Now you should be able to read larger mixed decimals, such as 48,567.2943, which is read forty-eight thousand, five hundred sixty-seven *and* two thousand nine hundred forty-three ten-thousandths.

Practice Problems

Here are some mixed decimals for you to read. Refer to the table if necessary. Write your answers in words and compare them with the answers at th back of this text.

1. 496.325
2. 2007.0126
3. 120,756.03250
4. 1,425,746.32472
5. 51.000342
6. 3.010203

Pointing Off Decimals

7. When you write a decimal, you write the number indicating the numerator first and then locate the decimal point so that you have as many decimal places as there are zeros in the denominator of the corresponding common fraction. You start counting decimal places at the *right-hand digit* and move toward the left. If there aren't enough digits in the numerator, you write zeros until you have the necessary number of places. This is called *pointing off the decimal.*

EXAMPLE 1. Write twenty-one hundredths as a decimal.

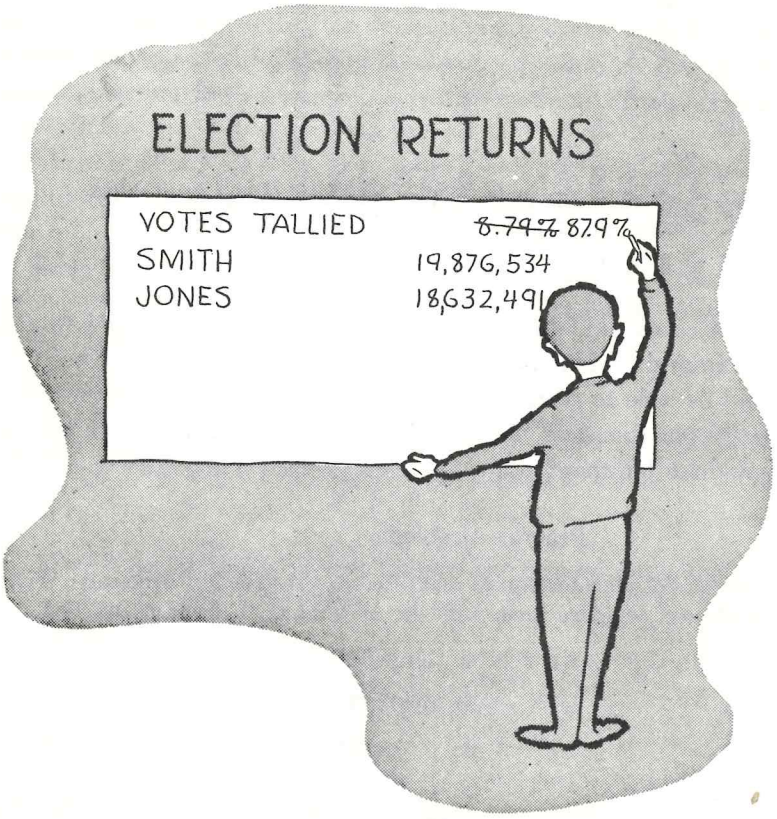
SOLUTION. The numerator is 21 and the denominator is 100, which has two zeros. Now there must be as many decimal places as there are zeros in the denominator; that is, there must be two decimal places. Since 21 has two places, we simply put the decimal point in front of the 2, thus getting .21. Ans.

EXAMPLE 2. Write thirty-nine ten-thousandths as a decimal.

SOLUTION. The numerator is 39 and the denominator is 10,000, which has four zeros. This means that there must be four decimal places. First we write 39. Now since this has only two decimal places, we must put two zeros in front of 39 to get the required four decimal places. Therefore, our answer must be .0039.

Importance of the Decimal Point

8. As we said earlier, the location of the decimal point in a number is very



important. Consider, for instance, the difference between \$5.00 and \$500 or between \$.05 and \$5. And note that it's the decimal point that makes the difference. In fact, the location of the decimal point is so vital that we can't stress it too much. Consider any whole number, for example 325, in which the decimal point does not appear. Even though the decimal point is not written, it must be understood to be to the right of 5. Later on, when you come to the basic arithmetic operations — addition, subtraction, multiplication, and division — you'll see that the proper placing of the decimal point is of the utmost importance.

Effect of Shifting Digit in Decimal

9. In reading whole numbers, you learned that the first digit at the *right* in a whole number represents units; the next digit to the left, tens; the next, hundreds; the fourth, thousands; and so on. Consider the number 4444, which may be written

$$\begin{array}{rr} & 4 \\ & 40 \\ & 400 \\ & 4000 \\ \text{sum} & 4444 \end{array}$$

The 4 at the left in 4444 denotes 4000, because it's in the thousands place. The next 4 denotes 400, because it's in the hundreds place; the third 4 denotes only 40, as it's in the tens place; and the last 4 denotes 4 units. You'll see at once that moving a 4 one place to the *right* reduces the number to one-tenth its former value. In the first position at the left, 4 denotes 4000; but the 4 in the next place to the right denotes only 400, or $\frac{1}{10}$ of 4000. The third 4 from the left denotes only 40, which is $\frac{1}{10}$ of 400; and the last 4 denotes 4 units, which is $\frac{1}{10}$ of 40. In other words, shifting the position of a digit *one place* to the *right* in a whole number divides its value by 10.

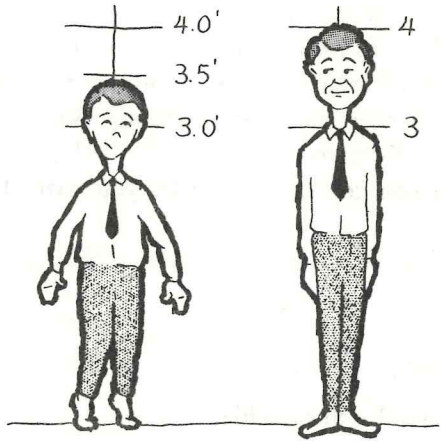
Let's see whether this applies to decimals too. Use the number 4444 as before, but add the decimal .4 to it, making 4444.4. The last 4 is one place to the right of the fourth 4, which is in the units place. Now, if shifting a digit one place to the right in a decimal divides its value by 10, the 4 following the decimal point should have a value of $\frac{1}{10}$ of 4 units, or $\frac{4}{10}$. We know that $.4 = \frac{4}{10}$; therefore, the statement is true for decimals as well as for whole numbers.

Cautions in Writing Decimals

10. If errors are to be avoided in writing decimals, certain facts must be kept in mind. For instance, inserting a zero between the decimal point and the first digit of the number divides the decimal by 10. Thus, if .5 is changed to .05 by the insertion of a zero, its value is divided by 10, because $.5 = \frac{5}{10}$ and $.05 = \frac{5}{100}$, and $\frac{5}{100}$ is $\frac{1}{10}$ of $\frac{5}{10}$.

All this can be summed up like this: *Moving the decimal point to the left in a number divides the number by 10 for each place that the point is moved.*

Taking a zero from the left of a decimal multiplies the decimal by 10. For example, take .0005, which has a value of $\frac{5}{10000}$. If one of the zeros is removed, the decimal becomes .005, which is equal to $\frac{5}{1000}$. Since $\frac{5}{1000} = 10 \times \frac{5}{10000}$, it's obvious that removing a zero has increased the value of the decimal 10 times.



Thus *moving the decimal point to the right multiplies the number by 10 for each place that the point is moved.*

But adding zeros to the right of a decimal or removing zeros from the right of a decimal has no effect on the value of the decimal. For example, adding a zero to .5 makes .50, but $.5 = \frac{5}{10}$ and $.50 = \frac{50}{100} = \frac{5}{10}$. Obviously, there has been no change in the value of the decimal. Similarly, if the zeros are dropped from .1200, it becomes .12. But $.12 = \frac{12}{100}$ and $.1200 = \frac{1200}{10000} = \frac{12}{100}$. Thus *removing or adding zeros at the right of a decimal does not change the value of the decimal.*

Check Your Learning

24. When a number does not have a written decimal point, the decimal point is understood to be at the (left, right) of the number.

(right)

25. Moving the decimal point one place to the left has the same effect as dividing the number by _____.

(10)

26. Dividing the mixed decimal 38.56 by 10 gives _____.

(3.856)

27. A number may be multiplied by 10 by moving the decimal point one place to the _____.

(right)

28. If the mixed decimal 23.4 is multiplied by 10, the result is _____.

(234)

Writing a Mixed Decimal as a Fraction

11. We've said that a number made up of a whole number and a decimal, such as 13.795, is a mixed decimal. Sometimes it's convenient to express a mixed decimal as a common improper fraction. Remember how you changed mixed numbers to improper fractions? Well, the procedure is the same for mixed decimals. First you write $13.795 = 13\frac{795}{1000}$. Then multiply 13 by 1000 and add 795, and you get 13,795. Finally you write 13,795 over the denominator 1000, to get $\frac{13795}{1000}$.

Here's another example: $2467.892 = 2467\frac{892}{1000}$ or $\frac{2467892}{1000}$.

Practice Problems

Write the following as decimals or as mixed decimals:

1. Twelve thousandths
2. Five *and* four tenths

3. Sixteen hundredths
4. Thirty-two ten-thousandths
5. Five hundred sixty-five millionths
6. Four hundred sixteen *and* two ten-thousandths
7. Five hundred six *and* five hundred six thousandths

The Four Basic Operations as Applied to Decimals

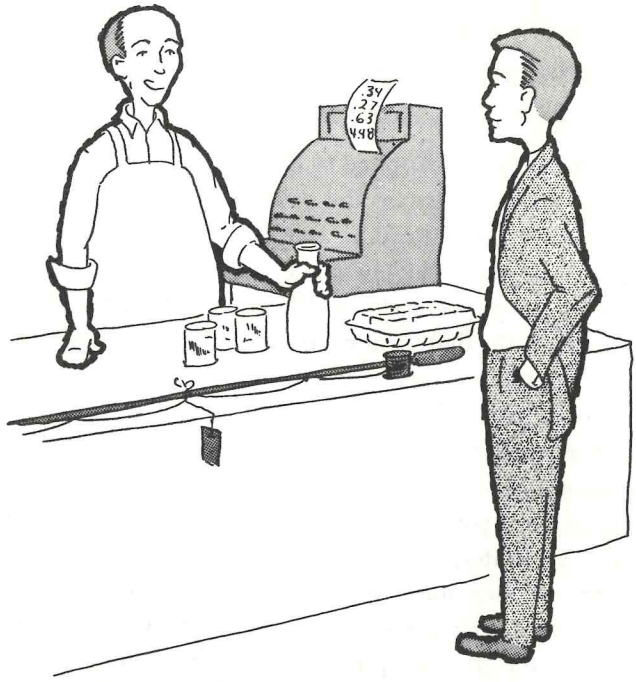
12. You're now ready to take up the study of the four fundamental operations – addition, subtraction, multiplication, and division – as they are applied to decimals. Once again your study will begin with a complete discussion of addition. However, before you begin your study of decimals, it's important that you realize you'll actually learn little that is new. In fact, the only new things you'll learn here are how to arrange decimals for adding, subtracting, multiplying, and dividing, and where to place the decimal point in the answer for each operation.

Now a word of caution: If you have any trouble adding, subtracting, multiplying, or dividing whole numbers, don't go any further with decimals. You're not ready to go on until you can add, subtract, multiply, and divide whole numbers quickly and accurately.

If you're slow or inaccurate in adding a column of numbers, go back and review the number combinations that were given in the practice problems for Part 1. A review of the practice problems can also help you in subtraction. For multiplication and division, you need to know the multiplication tables. If they're not as familiar to you as your ABC's, you'd better spend some time studying them right now. A little time spent in review now may save many hours of valuable study time later on.

We're quite sure that if you've followed our advice so far, you'll have no difficulty with decimals. We say this because we're sure that you'll:

1. Read every explanation carefully.
2. Study the solved examples until you fully understand each one.
3. Complete all statements in the Programmed Instruction sections in order to clarify important facts and principles.
4. Solve every practice problem so that you'll acquire the necessary skill.



Addition

Adding Decimals Only

13. Write down these numbers and add them: .35; .089; .5; and .0051. The correct answer is .9441. Is that what you got? If you didn't get this answer, you probably didn't arrange the decimal points in a vertical line. Your problem should look like this:

$$\begin{array}{r} .35 \\ .089 \\ .5 \\ .0051 \\ \hline .9441 \end{array}$$

The arrangement of the decimals is most important. The numbers must always be placed one below the other *with the decimal points in the same vertical line.* When this is done, tenths will be under tenths, hundredths under hundredths, and so on. You then add the decimals just as you'd add whole numbers. The decimal point in the answer is placed directly below the decimal points in the numbers you added.

Practice Problems

Add the following decimals.

1. .3 + .4 + .6 + .8 + .9
2. .34 + .8 + .316 + .079
3. .4 + .004 + .04 + .0004
4. .123 + .1 + .92 + .705 + .6
5. .03 + .198 + .56 + .0015 + .0196
6. .834 + .5796 + .8251 + .83975
7. .034 + .0056 + .000978 + .00008654
8. .91 + .347 + .82 + .9875
9. .864 +.547 + .821 + .3438 + .2
10. .246 + .3754 + .25 + .9 + .0875

Adding Whole Numbers and Decimals

14. You remember, of course, that when whole numbers are to be added, the right-hand digits are always arranged in a vertical line. Thus the sum of 14, 2, and 206 would be found as shown at the right. In adding whole numbers and decimals, however, the right-hand digits are not always in the same vertical line because the *decimal points* must be lined up. And as you've already learned, the decimal point for a whole number is understood. Therefore, when writing whole numbers and decimals in a column for addition, you should always write whole numbers with a decimal point so that you can line up all the decimal points. The following example will show you what we mean.

	14
	2
	206
sum	222

EXAMPLE. What is the sum of 242; 0.36; 118; 0.005; 6; and 0.1?

SOLUTION. First of all, we must write the whole numbers 242, 118, and 6 with decimal points. This will give us 242. and 118. and 6. The numbers to be added are then written one below the other *with the decimal points in the same vertical line,* as shown at the right.

Adding in the usual way, the sum is 366.465. Ans.

	242.
	0.36
	118.
	0.005
	6.
	0.1
sum	366.465

Practice Problems

Find the sum of each of these groups of decimals and whole numbers.

1. 32 + 0.32 + 8 + 16 + 0.594
2. 116 + 0.6 + 32 + 0.82 + 0.875

3. $349 + 0.75 + 0.25 + 54 + 0.097$
4. $0.06 + 0.9 + 13 + 0.32 + 0.874$
5. $0.9 + 1 + 82 + 365 + 0.787$
6. $40 + 8 + 0.856 + 2 + 0.021$
7. $690 + 0.35 + 0.5 + 70 + 0.555$
8. $4 + 0.3 + 75 + 0.16 + 253 + 0.824$
9. $5643 + 0.247 + 300 + 0.5 + 876 + 0.3356$
10. $0.25 + 0.176 + 0.4825 + 36 + 100 + 0.98$

Adding Decimals and Mixed Decimals

15. You've probably guessed by now that there's nothing new about the adding of decimals and mixed decimals. You're right! When adding decimals and mixed decimals, you write them one below the other, with the decimal points in a vertical line. Then you add them in the usual way, and place the decimal point in the answer directly under the decimal points in the numbers being added. Here's an example:

EXAMPLE. A bar is marked off into 8 parts, measuring 1.25, 4.31, 2.30, 7.6, 10.43, 5.56, 0.87, and 3.06 in. in length. What is the total length of the bar?

SOLUTION. The length of the bar is the sum of the lengths of the parts. The numbers are arranged so that the decimal points are in a vertical line, and are then added. This is illustrated in the work at the right.

The total length, therefore, is 35.38 in. Ans.

```
      1.25
      4.31
      2.30
      7.6
     10.43
      5.56
      0.87
      3.06
sum  35.38
```

Check Your Learning

29. When decimals are to be added, they must be arranged so that the decimal points are in the same _____ line.

(vertical)

30. After decimals have been arranged in a column, they are added in the same manner as _____ _____.

(whole numbers)

31. Whole numbers and decimals (can, cannot) be added.

(can)

32. When a whole number and a decimal are added, the whole number should be written with a _____ _____.

(decimal point)

33. The sum of 2 and 3.8 is _____ .

(5.8)

Practice Problems

Add each of the following groups of decimals and mixed decimals.

1. 35.3 + 0.4 + 0.03 + 50.25
2. 114.6 + 0.68 + 17.88 + 0.942
3. 88.02 + 0.35 + 44.5 + 60.15
4. 8.1 + 0.36 + 5.775 + 1.2345
5. 564.7 + 3.4 + 8965.25 + 0.2
6. 3489.1 + 257.3 + 4.864 + 5.009
7. 343.34 + 58.61 + 2.487 + 50.6
8. 3.6 + 0.01 + 40.05 + 10.82
9. 35.7 + 42.66 + 30.1 + 1.83
10. 25.84 + 736.54 + 259.77 + 565.891

Subtraction

How to Subtract Decimals

16. Subtraction of decimals, like subtraction of whole numbers and of fractions, involves only two numbers. As before, these numbers are called the *subtrahend* and the *minuend.* You remember, of course, that the subtrahend is the number to be subtracted and that the minuend is the number from which it is subtracted. Whether a problem in subtraction involves decimals, mixed decimals, or whole numbers, or any combination of these, you must be careful to keep the decimal points in a vertical line, just as you did in addition. The operation of subtraction is exactly the same in all details for decimals as it was for whole numbers, so review the basic principles for whole numbers if you're unsure of any part of subtraction.

And don't forget that the decimal point is placed in the answer, which is called the *difference,* directly beneath the decimal points in the minuend and subtrahend.

EXAMPLE 1. What is the difference between .786 and .208?

SOLUTION. The difference between any two numbers is found by subtracting the smaller number from the larger. Therefore, .208 will be the subtrahend because it is the smaller number and .786 will be the minuend. The work will look like this:

minuend	.786
subtrahend	.208
difference	.578

Be sure to notice that since the 8 in the subtrahend was larger than the 6 in the

minuend, it was necessary to borrow 1 from the preceding digit 8 in the minuend, making it 7. The 1 which was borrowed from the 8 changes the 6 to 16, and therefore you have $16 - 8 = 8$. The required difference is .578.

You've already seen that adding zeros to the right end of a decimal does not change the value of the decimal. And so, if either the subtrahend or the minuend has fewer decimal places than the other, we add zeros to the shorter number to give them the same number of decimal places. Let's consider an example in which the subtrahend is the shorter of the two.

EXAMPLE 2. Subtract .132 from .3063.

SOLUTION. The minuend has four decimal places while the subtrahend has only three. If we annex a zero to the subtrahend, the subtraction looks like this:

minuend	.3063
subtrahend	.1320
difference	.1743

The required difference is .1743. Ans.

Now let's consider an example in which the minuend has fewer decimal places than the subtrahend.

EXAMPLE 3. A rod 12 in. (inches) long has 2.75 in. cut from it. What length remains?

SOLUTION. To find the length of the rod that remains after a piece has been cut from it requires subtraction. The length remaining is the difference between 12 and 2.75. The whole number 12 has no decimal places, whereas 2.75 has two. We must, therefore, write the whole number as a mixed decimal having two decimal places. Thus it becomes 12.00. It is obvious that in all examples such as this, borrowing will be necessary. The actual subtraction goes like this:

minuend	12.00
subtrahend	2.75
difference	9.25

Hence the length remaining is 9.25 in. Ans.

Ordinarily, whenever zeros must be added, especially as in example 2, they are added mentally. And an experienced person might even omit writing the zeros in example 3. If you should decide not to show these zeros, be very sure that their presence is clearly understood.

Without the zeros the solutions for examples 2 and 3 would look like this:

.3063	
.132	
.1743	Ans.

12.	
2.75	
9.25	Ans.

Check Your Learning

34. In the subtraction of decimals, the two numbers are called the _____ and the _____ .

(subtrahend) (minuend)

35. The number to be subtracted is called the (minuend, subtrahend).

(subtrahend)

36. When decimals are subtracted, it (is, is not) necessary to keep the decimal point in a vertical line.

(is)

37. Adding zeros to the right of a decimal point (does, does not) change the value of the decimal.

(does not)

38. The decimal 365.8 is (larger than, smaller than, the same as) the decimal 365.80.

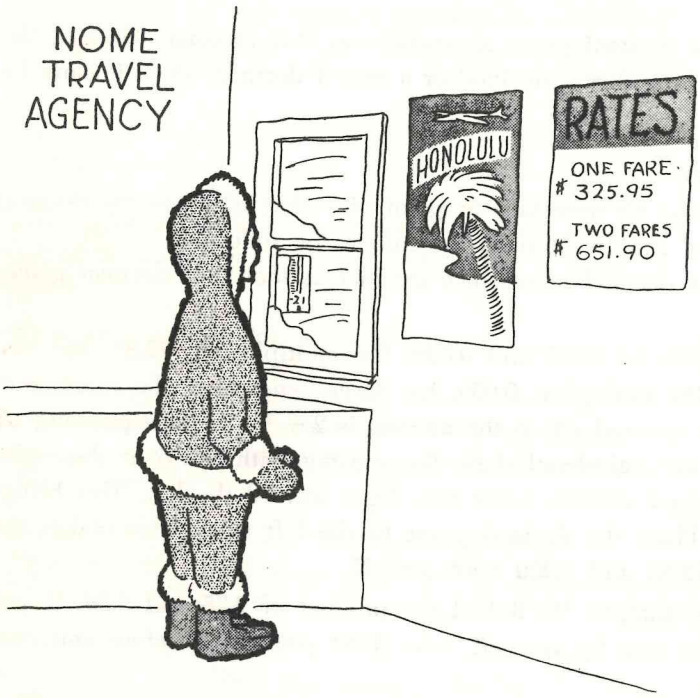
(the same as)

39. When there is a larger number of decimal places in the subtrahend than in the minuend, it is necessary to add the required number of _____ to the minuend.

(zeros)

Practice Problems

1. From 22.718 take 1.7042.
2. From 407.385 take 235.004.
3. Find the value of 22.718 — 1.778.
4. Subtract 6.5413 from 10.
5. Subtract 24 from 34.016.
6. From 50.014 take 1.012.
7. From 25.1 take 2.413.
8. Find the value of 20.001 — 19.999.
9. Subtract 1774.4 from 2351.86.
10. Subtract 0.212 from 25.02.
11. Subtract 0.91 from 20.
12. Find the difference if 40.1 is taken from 562.2.
13. From 2 take 0.0192.
14. From 25.62 take 1.7164.
15. Subtract 1774.2 from 25,613.



Multiplication

The Way to Multiply Decimals

17. Multiplication of decimals or of mixed decimals is performed in exactly the same way as multiplication of whole numbers. The two numbers to be multiplied are written one below the other so that the *right-hand* digits of both are in the same vertical column.

You do not line up the decimal points. In fact, you pay no attention to the position of the decimal points until the multiplication is complete. For example, if you were asked to multiply 72.85 by 0.039, your work would look like this:

multiplicand	72.85
multiplier	0.039
	65565
	21855
product	284115

As you can see, in this work we've ignored the decimal points and the number of decimal places in each factor. Nevertheless, it should be clear to you that 284115 isn't the correct answer. Even though you might not know just what the correct answer is, you should be able to see that 284115 cannot be the desired product. And why not? Simply because 284115 is the product of

the whole numbers 7285 and 39; it cannot be the product of 72.85 and 0.039. Right?

There has to be a decimal point somewhere in this product. That is, the correct product must be either a decimal or a mixed decimal, since it can't be a whole number.

Pointing Off a Product

18. You've seen that it's necessary to point off – that is, place the decimal point in – the product 284115. Here is the rule to be followed:

There are as many decimal places in a product as there are decimal places in both factors.

In the multiplication of 72.85 and 0.039, the multiplicand, 72.85, has two decimal places and the multiplier, 0.039, has three. Therefore, the number of decimal places to be pointed off in the answer is $2 + 3 = 5$. This pointing off *always* begins with the right-hand digit. So, starting with the 5 at the right-hand end of the product, 284115, count five digits toward the left. This brings you to the digit 8. Place the decimal point to the left of 8. This makes the correct product of 72.85 and 0.039 read 2.84115.

Let's try another example. We'll find the product of 2.75 and 3.26. If you think you can do this one by yourself, why don't you try it before you read any further?

Your work should look like this:

multiplicand	2.75
multiplier	3.26
	1650
	550
	825
product	8.9650

Since there are $2 + 2 = 4$ decimal places in both factors, we must have four decimal places in the answer. Counting four places to the left of the right-hand digit 0, we come to 9. We place the decimal point to the left of 9, and get 8.9650 as the correct answer.

At this stage you may be wondering whether the rule that there must be as many decimal places in the product as there are decimal places in both factors applies to all types of multiplication problems. "What happens," you may ask, "if one of the factors is a whole number?" The answer is: The rule applies at all times! Even when two whole numbers are multiplied together it applies; then the decimal places in the product are zero because there are no decimal places in either factor.

EXAMPLE 1. Multiply 0.825 by 13.

SOLUTION. In this example there are three decimal places in the multiplicand and none in the multiplier. Therefore, point off three decimal places in the product, beginning to count at the *right*. The product is 10.725. Ans.

multiplicand	0.825
multiplier	13
	2475
	825
product	10.725

EXAMPLE 2. What is the product of 426 and 0.005?

SOLUTION. In this example there are three decimal places in the multiplier and none in the multiplicand. Therefore, three decimal places are pointed off in the product, counting from the right. The product is 2.130, or 2.13. Ans.

multiplicand	426
multiplier	0.005
product	2.130

EXAMPLE 3. Multiply 1.205 by 1.15.

SOLUTION. In the multiplicand there are three decimal places and in the multiplier two decimal places, making five places in all to point off in the product, which gives 1.38575. Ans.

multiplicand	1.205
multiplier	1.15
	6025
	1205
	1205
product	1.38575

Pointing Off a Product Not Having Enough Digits

19. You won't try many problems in multiplication before you meet one where you have to point off more places in the product than you have digits. For example, if you multiply .325 by .015, you'll need six decimal places in the product. But when you multiply 325 by 15, you get 4875, which contains only four digits. "Then how do I point off six digits?" you may ask. The answer is simple. If the number of places to be pointed off is greater than the number of digits in the product, you *prefix zeros* to the product until you get the correct number of places. Note that to "prefix zeros" means to "put zeros *in front of*." You did much the same thing in example 2, Art. 7. Now let's look at some examples that illustrate this principle.

EXAMPLE 1. Multiply .076 by .012.

SOLUTION. There are three decimal places in the multiplicand and three in the multiplier, making six decimal places in all to be pointed off in the product. Since the product contains only three digits, we prefix three zeros to get the correct number of decimal places. The product is .000912. Ans.

multiplicand	.076
multiplier	.012
	152
	76
product	.000912

EXAMPLE 2. Multiply .232 by .001.

SOLUTION. In this example the multiplicand is multiplied by the digit 1 of the multiplier, which gives 232 as the product. The sum of the number of decimal places in the multiplicand and the multiplier is six, and so six places must be pointed off in the product. But as there are only three digits in the product, it is necessary to prefix three zeros, as shown, to get the required number of decimal places. Thus the product is .000232. Ans.

multiplicand	.232
multiplier	.001
product	.000232

Effect of Zeros Following Decimal Point

20. If a decimal or a mixed decimal being used as a multiplicand or a multiplier contains one or more zeros directly following the decimal point, as .0049 for example, such zeros have no effect on the digits forming the product. However, they do have an effect on the position of the decimal point in the product, since they must be counted to find the number of decimal places to be pointed off in the product.

Similarly, zeros at the end of a decimal or a mixed decimal may be dropped, because they have no effect on the value of the number. For example, 37.00 has the same value as 37, and .1750 is equal to .175.

EXAMPLE. Multiply .92500 by .313.

SOLUTION. As was just explained, the last zeros in the decimal .92500 may be omitted without affecting the product. Let's prove this by first multiplying the decimals in their original forms, as in solution (a), and then multiplying without the zeros in the multiplicand, as in solution (b).

In solution (a) there must be eight decimal places in the product. Eight is the sum of the decimal places in the multiplicand and multiplier. But if the two zeros are omitted from the multiplicand, as in (b), only six decimal places are required in the product. Notice that the two products are equal.

(a)

multiplicand	.92500
multiplier	.313
	277500
	92500
	277500
product	.28952500 Ans.

(b)

multiplicand	.925
multiplier	.313
	2775
	925
	2775
product	.289525 Ans.

Check Your Learning

40. When decimals are to be multiplied, it (is, is not) necessary to arrange the decimal points in a vertical line.

(is not)

41. Decimals are multiplied in the same manner as _____ _____.

(whole numbers)

42. After two decimals have been multiplied, it is necessary to point off the _____.

(product)

43. There are as many decimal places in the product as there are decimal places in both _____ combined.

(factors)

44. To point off a product, begin counting with the (left-hand, right-hand) digit.

(right-hand)

45. If the numbers 3.685 and 2.1 are multiplied, the product will have _____ decimal places.

(four)

Practice Problems

1. In the following examples find the product of:
   (*a*) 0.000492 × 4.1418
   (*b*) 4003.2 × 1.2
   (*c*) 0.021 × 0.002
   (*d*) 0.777 × 87.5
   (*e*) 6.72 × 2.222
   (*f*) 952.2 × 0.876
   (*g*) 7.7 × 257
   (*h*) 7.5000 × 252.7
2. Find the product of:
   (*a*) 0.176 and 0.197
   (*b*) 8.02 and 26
3. Multiply 42 times 0.124 times 0.001.
4. If 1 cu in. (cubic inch) of cast brass weighs 0.295 lb (pound), what is the weight of a brass casting containing 768 cu in.?
5. To obtain the weight of a connecting rod from a drawing, it was calculated that the rod contained 294.8 cu in. of wrought iron, 63.5 cu in. of brass, and 10.4 cu in. of babbitt. Assuming the weight of wrought iron to be 0.278 lb per cu in., that of brass 0.303 lb, and that of babbitt 0.264 lb, what was the weight of the rod? Express your answer to two decimal places.
6. The cost of manufacturing a certain article is $132. Of this amount, 0.6 is spent in the foundry, 0.3 on drilling, and the remainder on filing. What is the cost of each operation?

Division

Division and the Decimal Point

21. The methods used in dividing decimals are exactly the same as the ones used in dividing whole numbers. So be sure to review the division of whole numbers if you've forgotten any of the details. In the division of decimals, as in all the other operations involving decimals, by far the most important thing is the locating of the decimal point in the answer, or in this operation, the quotient. Again this is the only new thing you'll learn here.

The procedure for locating the decimal point in the quotient is quite simple. Let's begin with a division where the divisor is an integer, or a whole number. *When the divisor is an integer, the decimal point in the quotient is directly over the decimal point in the dividend.*

EXAMPLE 1. Divide 6.25 by 5.

SOLUTION. Divide just as you would if there were no decimal point in the dividend.

Then, after obtaining 125 in the quotient, put a decimal point in the quotient directly over the decimal point in the dividend. This gives 1.25 as the answer.

$$\begin{array}{rr} & 1.25 \quad \text{quotient} \\ \text{divisor} & 5\overline{)6.25} \quad \text{dividend} \end{array}$$

EXAMPLE 2. Divide 70.35 by 15.

SOLUTION.

$$\begin{array}{rr} & 4.69 \quad \text{quotient} \\ \text{divisor} & 15\overline{)70.35} \quad \text{dividend} \\ & \underline{60} \\ & 103 \\ & \underline{90} \\ & 135 \\ & \underline{135} \end{array}$$

EXAMPLE 3. Divide 3744.8 by 124.

SOLUTION.

$$\begin{array}{rr} & 30.2 \quad \text{quotient} \\ \text{divisor} & 124\overline{)3744.8} \quad \text{dividend} \\ & \underline{372} \\ & 248 \\ & \underline{248} \end{array}$$

EXAMPLE 4. Divide 0.625 by 25.

SOLUTION.

$$\begin{array}{rr} & 0.025 \quad \text{quotient} \\ \text{divisor} & 25\overline{)0.625} \quad \text{dividend} \\ & \underline{50} \\ & 125 \\ & \underline{125} \end{array}$$

In example 4, we had to put a zero in the quotient in the first decimal place after the decimal point because the divisor would not go into the first digit of the dividend. In the division of whole numbers, it wasn't necessary to show the zeros in the first part of the quotient. However, in the division of decimals, it is necessary to write the zeros because there must be as many decimal places in the quotient as there are in the dividend.

Before you go on to other divisions, you should be sure that you've mastered this particular type of division problem.

Practice Problems

Here are some problems to test your skill in division of decimals by whole numbers.

1. $2\overline{)1.94}$
2. $4\overline{).036}$
3. $13\overline{).962}$
4. $19\overline{)18.81}$
5. $7\overline{)7.28}$
6. $44\overline{)2.508}$
7. $51\overline{)55.59}$
8. $32\overline{)2.656}$
9. $9\overline{).5787}$
10. $8\overline{)4.24}$

22. *When the divisor is not an integer, we make it an integer by moving the decimal point to the right; at the same time, we move the decimal point in the dividend the same number of places to the right.* Changing the position of the decimal point in this way has no effect on the value of the quotient. You can easily verify this. Recall from your study of *Fractions* that the value of a fraction isn't changed if the numerator and the denominator are multiplied by the same number. For example,

$$\frac{13.5}{0.5} = \frac{13.5 \times 10}{0.5 \times 10} = \frac{135}{5}$$

$$\frac{17.44}{0.04} = \frac{17.44 \times 100}{0.04 \times 100} = \frac{1744}{4}$$

$$\frac{2.6412}{0.012} = \frac{2.6412 \times 1000}{0.012 \times 1000} = \frac{2641.2}{12}$$

This same work could be written like this:

$0.5\overline{)13.5} = 5\overline{)135}$ $\quad$ $0.04\overline{)17.44} = 4\overline{)1744}$ $\quad$ $0.012\overline{)2.6412} = 12\overline{)2641.2}$

Ordinarily, we don't rewrite problems in division like this. It's much simpler merely to *indicate the new position of the decimal point in the dividend.* Most people do this by inserting a caret (^) in the dividend where the decimal point should be. Thus the examples we've been using would be written:

$0.5\overline{)13.5}$ $\quad$ $0.04\overline{)17.44}$ $\quad$ $0.012\overline{)2.6412}$

and then we would divide just as we did when the divisor was an integer. For example:

$$0.5\overline{)13.5} \qquad 27.$$

$$0.04\overline{)17.44} \qquad 436.$$

$$0.012\overline{)2.6412} \qquad 220.1$$

EXAMPLE 1. Divide 56.7055 by 2.35.

SOLUTION. There are two decimal places in the divisor, and so we move the decimal point in the dividend two places to the right. We indicate the new position of the decimal point in the dividend by a caret. Then we divide, placing the decimal point in the quotient directly over the caret in the dividend.

```
                  24.13   quotient
divisor  2.35 ) 56.7055   dividend
                470  ^
                 970
                 940
                  305
                  235
                   705
                   705
```

EXAMPLE 2. Divide 10.53201 by 0.321.

SOLUTION. We move the decimal point in the dividend three places to the right because there are three decimal places in the divisor.

```
                  32.81   quotient
divisor  0.321 ) 10.53201  dividend
                 963  ^
                  902
                  642
                  2600
                  2568
                    321
                    321
```

EXAMPLE 3. Divide .42686 by .0014.

SOLUTION. The decimal point in the dividend is moved four places to the right.

```
                  304.9   quotient
divisor  .0014 ).42686    dividend
                 42   ^
                   68
                   56
                   126
                   126
```

EXAMPLE 4. Divide 0.00250 by 1.25.

SOLUTION.

```
                  0.002   quotient
divisor  1.25 ).00250     dividend
                    ^
```

EXAMPLE 5. Divide .125 by 0.005.

SOLUTION.

```
                  25.     quotient
divisor  0.005 ).125      dividend
                   ^
```

Practice Problems

1. $1.3\overline{)16.9}$
2. $.15\overline{).075}$
3. $.0162\overline{).86022}$
4. $7.82\overline{)0.0010166}$
5. $9.1\overline{)0.191373}$
6. $.027\overline{).13851}$
7. $0.231\overline{)74.151}$
8. $40.5\overline{)93.15}$

Annexing Zeros to Dividend

23. Very often the dividend has fewer decimal places than the divisor. When this happens, enough zeros are annexed, or added on, to the dividend so that we can move the decimal point to its proper position. If the dividend is a whole number and the divisor is a decimal, you'll have to add a decimal point and zeros to the whole number. A few examples will show you how to do this.

EXAMPLE 1. Divide 25.5 by 0.15.

SOLUTION. Since there is only one decimal place in the dividend, we must move the decimal point two places to the right. Therefore, we have to annex one zero to the dividend to provide the required number of decimal places. The quotient is 170. Ans.

```
                  170.    quotient
divisor  0.15 ) 25.50     dividend
                 15   ^
                 105
                 105
```

EXAMPLE 2. Divide 4935 by 0.235.

SOLUTION. Here we annex three zeros to the dividend because we must move the decimal point three places to the right. Then we divide in the regular way. The quotient is 21,000. Ans.

```
                    21000.   quotient
divisor  0.235 ) 4935.000    dividend
                  470   ^
                  235
                  235
```

EXAMPLE 3. Divide 133,216.64 by 1.0216.

SOLUTION.

```
                    130400.   quotient
divisor  1.0216 ) 133216.6400  dividend
                  10216   ^
                  31056
                  30648
                    40864
                    40864
```

The quotient is 130,400. Ans.

EXAMPLE 4. Divide 574 by 1.75.

SOLUTION.

```
                     328.   quotient
divisor  1.75 ) 574.00      dividend
                  525   ^
                   490
                   350
                   1400
                   1400
```

The quotient is 328. Ans.

Practice Problems

Here are some problems to test your understanding of the principles involved in the division of decimals.

1. $0.72\overline{)144}$
2. $0.02\overline{)765.4}$
3. $0.8\overline{)641.6}$
4. $0.013\overline{)277.03}$
5. $0.125\overline{)765}$
6. $0.023\overline{)621}$
7. $21.7\overline{)22.134}$
8. $423\overline{)4320.099}$

Dividing Decimals with a Remainder

24. In all the examples of the division of decimals you've had so far, the divisor was contained in the dividend without a remainder. In other words, the divisor went into the dividend evenly. The quotient obtained by such divisions is known as an exact decimal. We're sure you realize, however, that there will be times when divisions such as these will be the exception rather than the rule.

You learned in *Fractions* that if there is a remainder when one number is divided by another, the remainder can be made the numerator of a common fraction in which the denominator is the divisor. This fraction is annexed to the quotient. In working with decimals, the more usual procedure is to add zeros to the dividend and to continue the division until we have the required number

of decimal places in the quotient. As a matter of fact, this is one of the advantages of working with decimals rather than fractions. An example will show you what we mean.

EXAMPLE. What is the quotient of 199 divided by 15?

SOLUTION. In solution (*a*) the dividend and the divisor are whole numbers, and the remainder, 4, is used as the numerator in a fraction in which the divisor, 15, is the denominator. The quotient is a mixed number consisting of the whole number 13 and the fraction $\frac{4}{15}$, or $13\frac{4}{15}$.

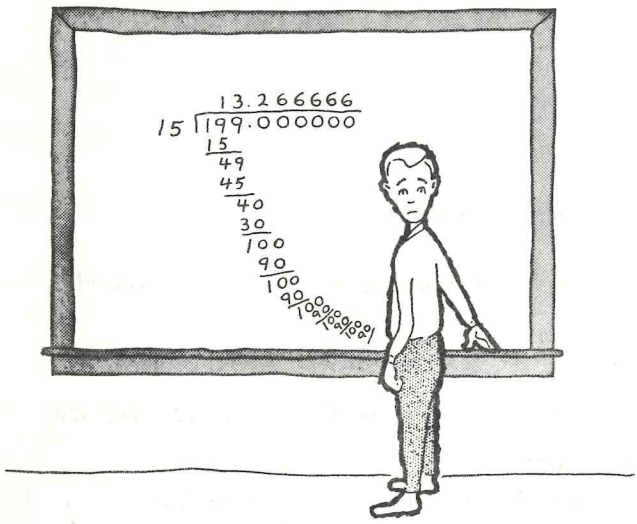
In solution (*b*), three zeros are added to the dividend and the division is continued as indicated.

(*a*)

$$\begin{array}{r} 13 + \frac{4}{15}, \text{ or } 13\frac{4}{15} \\ 15\overline{)199} \\ \underline{15} \\ 49 \\ \underline{45} \\ 4 \end{array}$$

(*b*)

$$\begin{array}{r} 13.266 \\ 15\overline{)199.000} \\ \underline{15} \\ 49 \\ \underline{45} \\ 40 \\ \underline{30} \\ 100 \\ \underline{90} \\ 100 \\ \underline{90} \\ \text{remainder } 10 \end{array}$$



What to Do When Division Does Not End

25. In the division of 199 by 15, you've seen two ways of expressing a quotient when the division has a remainder. In solution (*a*), the quotient $13\frac{4}{15}$ is an exact answer because $13\frac{4}{15} \times 15 = 199$. In solution (*b*), on the other hand, the quotient 13.266 is not an exact answer because $13.266 \times 15 = 198.990$. In fact, we could continue the division for many more decimal places, but the product of the quotient and the divisor would never equal the dividend. In this particular problem we'll always have a remainder of 10, no matter how far the division is carried.

Unfortunately, there's no completely satisfactory way of expressing an exact answer for solution (*b*). One suggestion is to use a fractional notation like this: $13.266\frac{2}{3}$. Now we get $13.266\frac{2}{3} \times 15 = 199$. But the big objection to using a decimal and a fraction like this is that it might be confusing. Another objection is that such a result is not as easily used in other calculations as if it were wholly a decimal.

In practical work, especially in engineering, it is preferable to use decimals entirely. Avoid fractions unless their use greatly simplifies the work. And do not use a fraction as part of a decimal. A person can seldom work with exact values in actual practice. He must therefore accept the fact that his results may not be exact. Still he *does* want his answers to be correct to a certain number of digits or to a certain number of decimal places.

In division, more often than not, we'll get answers that aren't exact. In other words, we'll have a remainder, and the quotient can be continued without limit. Such a result is a nonending, or *interminate, decimal.* Whenever you get an interminate decimal for an answer, you'll have to decide how many digits should be found. Then you'll know when to stop dividing.

More Facts About Decimals

Repeating Decimals

26. As you just learned, when the divisor is not contained evenly in the dividend, the division may be continued indefinitely. That is, the quotient may be an interminate decimal. In some instances, one or more figures of the quotient may be repeated. Thus, in dividing 1 by 6, the quotient is .1666 . . ., the digit 6 continuously repeating itself. This kind of decimal is known as a repeating decimal, or a *repeater.* The fact that a digit is being repeated is indicated sometimes by placing a dot over it. Thus the quotient .1666 . . . is written $.1\dot{6}$, indicating that 6 is a digit that will be repeated as the division is continued.

Circulating Decimals

27. Instead of only one digit of the quotient repeating itself, very often a series of two or more digits is repeated. For instance, the quotient of $1 \div 7$, by short division, is .142857142857. Here a whole group of digits is repeated. This group consists of the digits 1, 4, 2, 8, 5, and 7. This repeating decimal, sometimes called a *circulating decimal,* is written $.\dot{1}42857\dot{}$. The two dots, one above the 1 and the other above the 7, indicate the first and last digits of the repeating group of digits.

Check Your Learning

46. Zeros at the end of a decimal may be dropped because they have no effect on the _____ of the decimal.

(value)

47. The decimal point in a quotient is placed directly above the decimal point in the dividend when the divisor is a(n) _____ _____
(whole number)

48. In the division of decimals, it is necessary to write the zeros because there must be as many decimal places in the _____ as there are in the dividend.
(quotient)

49. When the divisor is a decimal, it must be changed to a whole number by moving the decimal point to the _____ .
(right)

50. When the divisor is a decimal and the dividend is a whole number, it is necessary to add as many _____ to the dividend as there are decimal places in the divisor.
(zeros)

51. If the divisor is not contained evenly in the dividend, the _____ may continue indefinitely.
(division)

Rounding Off Numbers

28. Let's imagine that you're asked to find the quotient of 271.13 ÷ 33 so that there will be two decimal places in the answer. You might think that the following work would give you the desired answer.

$$
\begin{array}{r}
8.21 \\
33\overline{)271.13} \\
264 \\
\hline
71 \\
66 \\
\hline
53 \\
33 \\
\hline
20
\end{array}
$$

However, if you consider this work carefully, you'll see that 8.21 is not the closest answer. Multiplying 33 by 8.21, you'll get 270.93, while the product of 33 and 8.22 is 271.26. Which of these is closer to 271.13? Obviously 271.26 is closer to 271.13 than is 270.93; therefore, 8.22 is a more nearly correct answer than is 8.21. Do you follow this reasoning? If you don't, please go back over it carefully, because it is a most important step in your learning.

This example shows the need for a statement of general procedure to be followed in this and similar problems. Here is what you do: First you must decide how many decimal places there are to be in the answer. Then you continue the

division for one more digit. This extra digit, which you'll discard later on, will be used to determine a more accurate value for the digit preceding it. If this extra digit is 5 or greater, you increase the preceding digit by 1, and then discard the extra digit. But if the extra digit is less than 5, you just drop it without making any change in the preceding digit. This procedure is called *rounding off.*

To illustrate: If we wanted four decimal places in the number 73.41425, we would express it as 73.4143. Here we discarded 5, so we increased the preceding digit, 2, by 1, making it 3. However, if we were interested in having only two decimal places, the number would be 73.41. This time we discarded a 4, so the digit preceding it wasn't changed.

Summary of Rounding Off

29. The entire procedure of rounding off demands very close study. Even though it seems quite simple to many students, it causes others a great deal of trouble. There are three important steps to remember.

1. Determine how many digits or decimal places are desired in the answer. In this text, to eliminate any doubt, we'll tell you exactly how many decimal places are to be found.
2. Continue the calculation to determine one digit more than is required.
3. Round off this result by dropping the extra digit. Add 1 to the preceding digit if the dropped digit is 5, 6, 7, 8, or 9. Do not change the preceding digit if the discarded digit is 0, 1, 2, 3, or 4.

This procedure is not limited to nonending decimals. We often round off when any decimal or whole number contains more digits than we want to use. Thus if we wanted to express $\frac{1}{32} = .03125$ as a decimal correct to four places, we'd give it as .0313. Likewise, if we wanted to express 34,743, in hundreds, or to the nearest hundred, we'd write 34,700. To the nearest thousand, 34,743 would be 35,000. Note that we've used the same rules for rounding off a whole number as for rounding off a decimal. In fact, the only new thing is *using a zero to take the place of the digits we discarded.* Be sure to do this, or sometime you may make the ridiculous statement that 34,743 rounds off to 35.

Check Your Learning

52. In rounding off a number, when the extra digit is 5 or more, _____ the preceding digit by 1.

(increase)

53. To round off a number when the extra digit is less than 5, simply _____ the extra digit without changing the preceding one.

(drop)

54. Rounding off the decimal .7645 to three decimal places gives _____.
(.765)

55. When you round off the decimal .7645 to two decimal places your answer is _____.
(.76)

56. If a digit is dropped in rounding off a whole number, a _____ must be used to replace it.
(zero)

57. Rounding off the number 4621 to the nearest hundred gives _____.
(4600)

58. The decimal .0569 rounded off to three decimal places is _____.
(.057)

Reducing a Fraction to a Decimal

30. Common fractions must often be changed to decimals. One of the most familiar examples of such reduction is the changing of fractions of an inch to equivalent decimals of a foot. The procedure is a simple one. *To reduce a common fraction to a decimal, place a decimal point after the numerator, annex zeros to the right of the decimal point, and divide by the denominator.*

EXAMPLE 1. Find the decimal that is equivalent to the fraction $\frac{3}{4}$.

SOLUTION. By short division, the solution is as shown. Only two zeros were annexed to the numerator because at that point the division ended.

$$4\overline{)3.00}\quad 0.75$$

or $\frac{3}{4} = 0.75$ Ans.

EXAMPLE 2. What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.

$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ \underline{64} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \end{array}$$

Here again the division ends. Therefore the answer is $\frac{7}{8} = .875$. Expressed to two decimal places, $\frac{7}{8} = .88$.

EXAMPLE 3. What decimal is equivalent to $\frac{4}{11}$?

SOLUTION. With the division performed as shown, it is found that there is a remainder and that the division may be carried on indefinitely. It is also seen that the quotient is a repeating decimal.

$$
\begin{array}{r}
0.3636 \\
11\overline{)\,4.0000} \\
\underline{33} \\
70 \\
\underline{66} \\
40 \\
\underline{33} \\
70 \\
\underline{66} \\
4
\end{array}
$$

or $\frac{4}{11} = 0.3636$. Ans.

Note that if the answer were desired to three decimal places, it would be $\frac{4}{11} = 0.364$.

Check Your Learning

59. A common fraction (can, cannot) be changed to a decimal.

(can)

60. A common fraction is changed to a decimal by dividing the _____ into the _____.

(denominator) (numerator)

61. When dividing the numerator of a fraction by the denominator, you often have to add a decimal point and a certain number of _____ to the numerator.

(zeros)

62. To find the decimal equivalent of $\frac{6}{7}$, it is necessary to divide _____ into _____.

(7) (6)

Reducing a Decimal to a Fraction

31. As we said before, you'll be more interested in working with decimals than with fractions. However, after seeing how a fraction can be converted to a decimal, you'll probably wonder for the moment how a decimal can be converted to a fraction. We're sure you'll remember the explanation given in Art. 3. The only new thing to be considered is reduction of the fraction to its lowest terms by dividing both numerator and denominator by the same number. Be sure to go back and read Art. 3 carefully. Then study these examples.

EXAMPLE 1. Reduce .125 to a fraction.

SOLUTION. $.125 = \frac{125}{1000} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8}$. Ans.

EXAMPLE 2. Reduce .875 to a fraction.

SOLUTION. $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$. Ans.

Reducing a Decimal to a Fraction with a Specified Denominator

32. You'll often want to reduce a decimal to a fraction having a certain denominator. When you do, multiply the decimal by the desired denominator. This

will give you the numerator of the fraction. For example, suppose that you want to change .375 to a fraction with 8 as the denominator. To find the numerator, multiply .375 by 8. Since this product is 3.000, or 3, the required fraction is $\frac{3}{8}$. Thus $.375 = .375 \times \frac{8}{8} = \frac{3}{8}$.

As another example, suppose that you want to change .667 to a fraction with 16 as the denominator. Find the numerator by multiplying .667 by 16. This product is 10.672. Since 10.672 rounded off to a whole number is 11, the fraction is $\frac{11}{16}$. Never say that the fraction is $\frac{10.672}{16}$. This last example shows what you must do whenever the numerator of the fraction is not a whole number. You'll often get a mixed decimal as the numerator of the desired fraction. When this happens, round off the numerator to a whole number.

Check Your Learning

63. When a decimal is changed to a common fraction, the numerator of the fraction is made up of the digits in the decimal written without a _____ _____.

(decimal point)

64. When a decimal is changed to a common fraction, the denominator of the fraction is 1 followed by as many _____ as there are decimal places in the decimal.

(zeros)

65. Changing the decimal .67 to a common fraction, we have _____.

($\frac{67}{100}$)

66. After a decimal has been changed to a common fraction, the fraction must be _____ to its lowest terms.

(reduced)

67. To change a decimal to a fraction having a certain denominator, multiply the decimal by the desired denominator to produce the _____ of the fraction.

(numerator)

68. Whenever the numerator of a fraction is a decimal, round the decimal off to a _____ _____.

(whole number)

Stop, Think, and Review

33. Before you begin work on the examination questions, take a few minutes to think about the material covered in this text. Is there any section of the text that isn't perfectly clear to you? If so, GO BACK AND REVIEW.

Answers to Practice Problems

Article 3

1. $\frac{3}{10}$
2. $\frac{8}{10}$, or $\frac{4}{5}$
3. $\frac{14}{100}$, or $\frac{7}{50}$
4. $\frac{816}{1000}$, or $\frac{102}{125}$
5. $\frac{304}{1000}$, or $\frac{38}{125}$
6. $\frac{225}{1000}$, or $\frac{9}{40}$
7. $\frac{349}{1000}$
8. $\frac{4125}{10000}$, or $\frac{33}{80}$
9. $\frac{2843}{10000}$
10. $\frac{5698}{10000}$, or $\frac{2849}{5000}$
11. $\frac{25946}{100000}$, or $\frac{12973}{50000}$
12. $\frac{309468}{1000000}$, or $\frac{77367}{250000}$

Article 4

1. $\frac{8}{10}$ Eight tenths
2. $\frac{25}{100}$ Twenty-five hundredths
3. $\frac{625}{1000}$ Six hundred twenty-five thousandths
4. $\frac{4876}{10000}$ Four thousand eight hundred seventy-six ten-thousandths
5. $\frac{25843}{100000}$ Twenty-five thousand, eight hundred forty-three hundred-thousandths
6. $\frac{8732845}{10000000}$ Eight hundred seventy-three thousand, two hundred forty-five millionths

Article 6

1. Four hundred ninety-six *and* three hundred twenty-five thousandths
2. Two thousand seven *and* one hundred twenty-six ten-thousandths
3. One hundred twenty thousand, seven hundred fifty-six *and* three thousand two hundred fifty hundred-thousandths
4. One million, four hundred twenty-five thousand, seven hundred forty-six *and* thirty-two thousand four hundred seventy-two hundred thousandths
5. Fifty-one *and* three hundred forty-two millionths
6. Three *and* ten thousand two hundred three millionths

Article 11

1. .012
2. 5.4
3. .16
4. .0032
5. .000565
6. 416.0002
7. 506.506

Article 13

1. 3
2. 1.535
3. .4444
4. 2.448
5. .8091
6. 3.07845
7. .04066454
8. 3.0645
9. 2.7758
10. 1.8589

Article 14

1. 56.914
2. 150.295
3. 404.097
4. 15.154
5. 449.687
6. 50.877
7. 761.405
8. 333.284
9. 6820.0826
10. 137.8885

Article 15

1. 85.98
2. 134.102
3. 193.02
4. 15.4695
5. 9533.55
6. 3756.273
7. 455.037
8. 54.48
9. 110.29
10. 1588.041

Article 16

1. 21.0138
2. 172.381
3. 20.94
4. 3.4587
5. 10.016
6. 49.002
7. 22.687
8. 0.002
9. 577.46
10. 24.808
11. 19.09
12. 522.1
13. 1.9808
14. 23.9036
15. 23,838.8

Article 20

1. *a*) .0020377656
   b) 4803.84
   c) .000042
   d) 67.9875
   e) 14.93184
   f) 834.1272
   g) 1978.9
   h) 1895.25
2. *a*) .034672
   b) 208.52
3. .005208
4. 226.56 lb
5. 103.94 lb
6. $79.20 foundry
   $39.60 drilling
   $13.20 filing

Article 21

1. .97
2. .009
3. .074
4. .99
5. 1.04
6. .057
7. 1.09
8. .083
9. .0643
10. .53

Article 22

1. 13
2. .5
3. 53.1
4. .00013
5. .02103
6. 5.13
7. 321
8. 2.3

Article 23

1. 200
2. 38,270
3. 802
4. 21,310
5. 6120
6. 27,000
7. 1.02
8. 10.213

Practical Arithmetic

Serial 6750C PART 3 Edition 1

Examination Questions

NOTICE TO STUDENTS.—Study this instruction text thoroughly before you try to answer the following questions. Read each question carefully and be sure you understand it. Then write the best answer that you can. If your answer requires a mathematical solution, show enough of your work to indicate the method you used. Unless you do this we will have no way of knowing how you obtained your answer and your work will not be considered complete. After you have answered all the questions, double-check them to make sure that this is the best work you can do. Don't omit any questions.

Mail your work to us as soon as it is finished. DON'T HOLD IT until another examination is ready.

Each of the following questions is worth 10 points.

1. Find the sum of each of the following groups of numbers.
   a) 0.07; 16.03; 0.45
   b) 2.68; 14.5; 0.06; 24.31
   c) 326.5; 29.9; 207.06; 0.003
   d) 1000; 200.50; 2.3867; 16.409

2. Determine the difference between the following numbers.
   a) 0.6791 and 0.238
   b) 21.3852 and 20.4986
   c) 48 and 0.0036
   d) 407.2 and 22

3. Write each of the following numbers first as a decimal and then as a common fraction.
   a) Nine tenths
   b) Four hundred twenty-five thousandths
   c) Sixteen ten-thousandths
   d) Two and twenty-three thousandths

4. Express:
   a) 0.25 as a fraction having a denominator of 20.
   b) 0.1416 in thirty-seconds.
   c) 0.4 as a fraction with a denominator of 15.
   d) 0.364 as a fraction with a denominator of 8.

5. Find the product of the following numbers. (Work required on *c* and *d*)
   a) 368 and 0.2
   b) 27.65 and 0.007
   c) 0.173 and 0.504 (Round off your answer to four decimal places.)
   d) 2.7 and 21 and 0.32 (Round off your answer to two decimal places.)

6. Find the quotient of
   a) 3045 ÷ 0.5
   b) 241.92 ÷ 3.2 (Work required on *b*, *c*, and *d*)
   c) 0.7854 ÷ 420
   d) 8.36 ÷ 18.42 (Express your answer to two decimal places.)

7. Round off each of the following numbers to two decimal places.
   a) 38.2589
   b) 369.7048
   c) 0.28336
   d) 0.0675

8. Change the following fractions to decimal form.
   a) $\frac{7}{8}$
   b) $\frac{4}{5}$
   c) $\frac{1}{2\frac{3}{3}}$ (Round off your answer to three decimal places.)
   d) $\frac{17}{3}$ (Round off your answer to two decimal places.)

9. If 0.026 in. of nickel plate is deposited per hour, what is the thickness deposited in $8\frac{3}{4}$ hr? Give your answer
   a) as a decimal.
   b) as a fraction in its simplest form.
   c) expressed to three decimal places.

10. A machinist measures the diameter of a shaft with a micrometer. The reading he obtains is 2.0446 in. Give this measurement
    a) rounded off to three decimal places.
    b) rounded off to two decimal places.
    c) rounded off to one decimal place.
    d) as a whole number.